

Complementary Filters Shaping Using \mathcal{H}_∞ Synthesis

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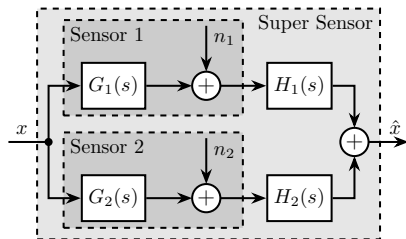
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Motivation - Sensor Fusion Architecture



$$\hat{x} = (G_1H_1 + G_2H_2)x + H_1n_1 + H_2n_2$$

Complementary Property

$$H_1(s) + H_2(s) = 1$$

Let's first consider **Perfectly Known Sensor Dynamics**:

$$G_1(s) = G_2(s) = 1 \implies$$

$$\hat{x} = x + H_1n_1 + H_2n_2$$

We need a synthesis method of complementary filter that allows to **shape the norm of the generated filters**.

Shaping of Complementary Filters using \mathcal{H}_∞ synthesis

Design Objective

$$H_1(s) + H_2(s) = 1$$

$$|H_1(j\omega)| \leq \frac{1}{|W_1(j\omega)|} \quad \forall \omega$$

$$|H_2(j\omega)| \leq \frac{1}{|W_2(j\omega)|} \quad \forall \omega$$

\mathcal{H}_∞ Synthesis

Find $H_2(s)$ such that:

$$\left\| \begin{bmatrix} [1 - H_2(s)] W_1(s) \\ H_2(s) W_2(s) \end{bmatrix} \right\|_\infty \leq 1$$

$$H_1(s) \triangleq 1 - H_2(s)$$

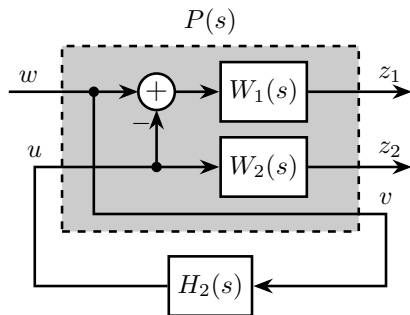


Figure: \mathcal{H}_∞ Architecture used for the shaping of complementary filters

Results - \mathcal{H}_∞ Synthesis of Complementary filters

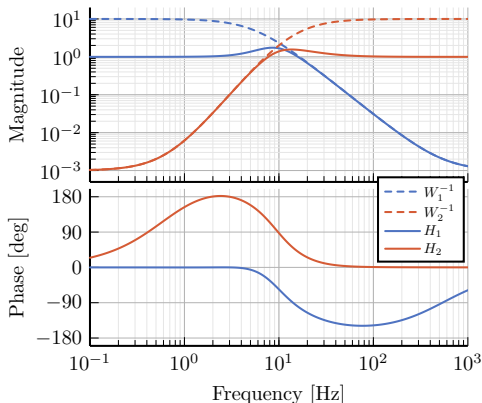



Figure: Frequency response of the weighting functions and complementary filters obtained using \mathcal{H}_∞ synthesis


Poster TF1-114



COMPLEMENTARY FILTERS SHAPING USING \mathcal{H}_∞ SYNTHESIS

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Abstract We present synthesis, key feasibility and feature aspects regarding to several systems when using an optimal linear complementary filter. The paper presents a method for shaping this complementary filter, by the synthesis that allows to shape the filter shape. The method is shown to be easily applicable for the synthesis of complex complementary filter.

Introduction
 Complementary filters are used often in control systems to reconstruct the state quantities without using direct measurements. Typically, integration of each output is filtered and decomposed to form a single sensor giving a better estimate over a wider bandwidth. The technique is called sensor fusion and is widely used in applications ranging from the attitude estimation of UAVs to the motion control of the MEMS IMUs. In the paper, sensor decomposition is applied to the complementary filter using the optimal design approach. This paper has been presented at the conference "IFAC on model-based design methods to design the control of the complementary filters available. The author's research project is not a part of the IFAC system.

Sensor Fusion Architecture
 Let us consider two sensors measuring the same physical quantity x with dynamics $D_1(s)$ and $D_2(s)$, and with associated noise characteristics w_1 and w_2 . The signals from both sensors are fed into the complementary filters $H_1(s)$ and $H_2(s)$ and their outputs are then summed using the sum $H(s)$ as a constraint $H = H_1 + H_2$.
 The complementary property of $H_1(s)$ and $H_2(s)$ implies: $H_1(s) + H_2(s) = 1$.
 For the transfer functions to be equal to 1 at all frequencies:

Complementary Filters Requirements
Noise Property
 First sensor partially losses sensor bandwidth and that it is not necessary.
 The structure is then:

$$H_1(s) = \frac{D_1(s)}{D_1(s) + D_2(s)}$$
 The structure is then:

$$H_2(s) = \frac{D_2(s)}{D_1(s) + D_2(s)}$$
 The structure is then:

$$H(s) = \frac{D_1(s) + D_2(s)}{D_1(s) + D_2(s)}$$
 The structure is then:

$$H(s) = \frac{D_1(s) + D_2(s)}{D_1(s) + D_2(s)}$$
 The structure is then:

$$H(s) = \frac{D_1(s) + D_2(s)}{D_1(s) + D_2(s)}$$

Dynamical Uncertainty
 Let us consider a system consisting in multiplicative input uncertainty: $G(s) = G_0(s)(1 + \Delta(s))$.
 The structure is then:

$$H_1(s) = \frac{G_0(s)}{G_0(s) + G_0(s)\Delta(s)}$$
 The structure is then:

$$H_2(s) = \frac{G_0(s)\Delta(s)}{G_0(s) + G_0(s)\Delta(s)}$$
 The structure is then:

$$H(s) = \frac{G_0(s) + G_0(s)\Delta(s)}{G_0(s) + G_0(s)\Delta(s)}$$

Conclusion
 Complementary filters can be used to combine multiple sensors to obtain a single sensor. Identification of the noise characteristics and the robustness of the filter structure are important aspects to be taken into account in the synthesis of the complementary filter. The method that permits the shaping of the complementary filter structure has been proposed and has been successfully applied to the design of complex filters.

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